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Phenomenological Evidence for the Gluon Content of η and η'

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Abstract:

We discuss the effect of η - η' mixing and of axial current anomalies on various decay processes. We describe the 2γ decays of η and η' , the radiative decays of low-lying vector mesons as well as the decays $\eta \rightarrow 3\pi$, $J/\psi \rightarrow \eta\gamma$ and $J/\psi \rightarrow \eta'\gamma$ in one unified framework, in which they are related to the electromagnetic and strong axial current anomalies, respectively. We also discuss briefly the enhancement of η' in D_s decays and suggest a possible relation to gluon-mediated processes. The importance of the decays of the glueball candidates $f_0(1500)$ and $f_0(1590)$ into $\eta\eta'$ channels is also stressed.

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1. In this paper we study the effect of η - η' mixing and of the axial current anomaly on various processes. We show that a consistent picture arises for the radiative decays $\eta \rightarrow 2\gamma$ and $\eta' \rightarrow 2\gamma$, for radiative decays of type $V \rightarrow \eta\gamma$, $\eta' \rightarrow V\gamma$ of the lowest-lying vector mesons V , as well as for $\eta \rightarrow 3\pi$ and $J/\psi \rightarrow \eta(\eta')\gamma$. Whereas for all the other decays we follow closely the approach of Ref. [1], our description of the vector meson decays is based on their relation to the QED triangle anomaly. In Sec. 2 we thus first test our approach for the simpler cases of the decays $V \rightarrow \pi(K)\gamma$. In Sec. 3 we then establish notations for the η - η' system and determine the decay constants f_0 and f_8 as functions of the mixing angle from the 2γ decays of η and η' . Section 4 is devoted to the comparison of our expectations for the widths of $V \rightarrow \eta\gamma$, $\eta' \rightarrow V\gamma$ with experiment, whereas Sec. 5 deals with the decay $\eta \rightarrow 3\pi$, which probes the light quark content of η . In Sec. 6 we then investigate the gluon content of η and η' , which determines the decays $J/\psi \rightarrow \eta\gamma$ and $J/\psi \rightarrow \eta'\gamma$. It is in this channel that our approach yields a dependence of the involved matrix elements on the mixing angle that is significantly different from other models. We briefly comment on this in relation to the approach of Veneziano et al., Ref. [2], and close in Sec. 7 with some remarks on the important rôle of anomalies in other gluon-enriched channels like $D_s \rightarrow \eta(\eta')X$ and in glueball searches.

2. The radiative decays of the lowest-lying vector-meson nonet are traditionally described in terms of magnetic moments of quarks, see e.g. [3], or of unknown couplings related by $SU_F(3)$ symmetry [4]. In Refs. [5, 6] the width $\Gamma(\rho \rightarrow \pi\gamma)$ was related via $SU_F(3)$ arguments and vector-meson dominance to the radiative width of π . In this paper we relate the radiative decays of light vector mesons to light pseudoscalars, $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$, directly to the anomaly of the AVV triangle diagram, where A stands for an axial-vector and V for a vector current. Our approach¹ both includes $SU_F(3)$ breaking effects and fixes the vertex couplings $g_{VP\gamma}$ as defined below².

Let us start by considering the correlation function

$$i \int d^4x e^{iq_2x} \langle \pi(q_1 + q_2) | T J_\mu^{\text{EM}}(x) J_\nu^{I=0(1)}(0) | 0 \rangle = \epsilon_{\mu\nu\rho\sigma} q_1^\rho q_2^\sigma F_{\pi,I=0(1)}(q_1^2, q_2^2), \quad (1)$$

with the currents

$$J_\mu^{\text{EM}} = \frac{2}{3} \bar{u}\gamma_\mu u - \frac{1}{3} \bar{d}\gamma_\mu d - \frac{1}{3} \bar{s}\gamma_\mu s, \quad J_\mu^{I=0,1} = \frac{1}{\sqrt{2}} (\bar{u}\gamma_\mu u \pm \bar{d}\gamma_\mu d). \quad (2)$$

The values of $F_{\pi,I}(0,0)$ are fixed by the QED triangle anomaly as

$$F_{\pi,I=1}(0,0) = \frac{1}{4\pi^2 f_\pi}, \quad F_{\pi,I=0}(0,0) = \frac{3}{4\pi^2 f_\pi}, \quad (3)$$

where the pion leptonic decay constant f_π is defined as

$$\langle 0 | \bar{u}\gamma_\mu \gamma_5 d | \pi^- \rangle = i f_\pi p_\mu, \quad (4)$$

¹A similar approach was proposed in Ref. [7].

²The behaviour of the anomalous contribution has been studied quite extensively, in particular for large space-like momenta, and found to fall off asymptotically like $1/-q^2$ [8]. The results of this section show however that in the physical sector, and for momenta up to the square of the ϕ mass, the full value of the anomaly gives sensible results. It was important for us to check this fact in the ρ, ω, π, K sector, independently of further difficulties associated with the strong anomaly.

so that $f_\pi = 0.132 \text{ GeV}$ from $\pi \rightarrow \mu\nu_\mu$. For the analogously defined kaon decay constant one finds $f_K = 0.160 \text{ GeV}$. Using their analytic properties, we can express these form factors by a dispersion relation in the momentum of the isospin current³:

$$F_{\pi,I=1(0)}(0,0) = \frac{1}{\pi} \int_{4(9)m_\pi^2}^{\infty} ds \frac{1}{s} \text{Im} F_{\pi,I=1(0)}(s,0). \quad (5)$$

Saturating by the lowest-lying resonances we obtain

$$F_{\pi,I=1}(0,0) = \frac{f_\rho}{m_\rho} g_{\rho\pi\gamma} + \dots, \quad F_{\pi,I=0}(0,0) = \frac{f_\omega}{m_\omega} g_{\omega\pi\gamma} + \dots \quad (6)$$

Here the dots stand for higher resonances and multi-particle contributions to the correlation function. In the following we assume vector meson dominance and thus neglect these contributions. Although this assumption may be criticized (and actually in Ref. [6] the continuum contributions were modelled by the perturbative spectral function above some threshold, but without taking into account the motion of the light quarks in the pion), it yields a description of the data that is good enough for our purposes⁴.

The f_V in (6) are the vector mesons' leptonic decay constants defined by

$$\langle 0 | J_\mu^V | V(p, \lambda) \rangle = m_V f_V \epsilon_\mu^{(\lambda)}(p). \quad (7)$$

λ denotes the helicity state of the meson. For ideal mixing the relevant currents are $J_\mu^\rho = J_\mu^{I=1}$, $J_\mu^\omega = J_\mu^{I=0}$ and $J_\mu^\phi = -\bar{s}\gamma_\mu s$. In the following we include the deviation from ideal mixing by taking into account a mixing-angle $\theta_V = 40.3^\circ$ for ϕ - ω mixing⁵. The f_V can be determined from the experimental decay rates [9] via

$$\Gamma(V \rightarrow e^+ e^-) = c_V \pi \alpha^2 \frac{f_V^2}{m_V} \quad (8)$$

with $c_V = \{2/3, 2/9 \sin^2 \theta_V, 2/9 \cos^2 \theta_V\}$ for $V = \{\rho^0, \omega, \phi\}$. The experimental results are

$$f_{\rho^0} = (216 \pm 5) \text{ MeV}, \quad f_\omega = (174 \pm 3) \text{ MeV}, \quad f_\phi = (254 \pm 3) \text{ MeV}. \quad (9)$$

The charged mesons decay constants can be obtained from $\tau \rightarrow V\nu_\tau$ via

$$\Gamma(\tau^- \rightarrow V\nu_\tau) = \frac{G_F^2 |V_{ij}|^2}{16\pi} f_V^2 m_\tau^3 \left(1 - \frac{m_V^2}{m_\tau^2}\right)^2 \left(1 + 2 \frac{m_V^2}{m_\tau^2}\right), \quad (10)$$

where V_{ij} is the appropriate CKM matrix element. We find

$$f_{\rho^\pm} = (195 \pm 7) \text{ MeV}, \quad f_{K^{*\pm}} = (226 \pm 28) \text{ MeV}. \quad (11)$$

In view of the discrepancies between the measurements of the ρ decay constant from the charged and neutral sectors, we will use the average value $f_\rho = (205 \pm 17) \text{ MeV}$ in the following.

³Note that the dispersion relation needs no subtraction, and that the value $F_{\pi,I}(0,0)$ is unambiguously fixed by the anomaly.

⁴We are interested here in understanding how the gluon anomaly affects decays involving η and η' and not so much in a detailed fit of radiative vector meson decays.

⁵This value follows from the standard $\text{SU}_F(3)$ breaking analysis; the ideal mixing angle is given by $\tan \theta_V = 1/\sqrt{2}$, corresponding to $\theta_V = 35.3^\circ$.

P	V	$g_{VP\gamma}$ (th.)	$g_{VP\gamma}$ (exp.)
π^\pm	ρ^\pm	$\frac{m_\rho}{4f_\pi f_\rho \pi^2} = (0.73 \pm 0.07) \text{ GeV}^{-1}$	$(0.74 \pm 0.04) \text{ GeV}^{-1}$
π^0	ρ^0	$\frac{m_\rho}{4f_\pi f_\rho \pi^2} = (0.73 \pm 0.07) \text{ GeV}^{-1}$	$(0.98 \pm 0.13) \text{ GeV}^{-1}$
π	ω	$\frac{3m_\omega}{4f_\pi f_\omega \pi^2} \frac{\sin \theta_V + \sqrt{2} \cos \theta_V}{\sqrt{3}} = (2.58 \pm 0.04) \text{ GeV}^{-1}$	$(2.33 \pm 0.07) \text{ GeV}^{-1}$
π	ϕ	$\frac{\sqrt{3}m_\phi}{4f_\pi f_\phi \pi^2} (\sqrt{2} \sin \theta_V - \cos \theta_V) = (0.20 \pm 0.01) \text{ GeV}^{-1}$	$(0.14 \pm 0.01) \text{ GeV}^{-1}$
K^0	\bar{K}^{0*}	$\frac{m_{K^*}}{2f_{K^*} f_K \pi^2} = (1.25 \pm 0.15) \text{ GeV}^{-1}$	$(1.25 \pm 0.05) \text{ GeV}^{-1}$
K^\pm	$\bar{K}^{\pm*}$	$\frac{m_{K^*}}{4f_{K^*} f_K \pi^2} = (0.62 \pm 0.08) \text{ GeV}^{-1}$	$(0.84 \pm 0.04) \text{ GeV}^{-1}$

Table 1: Theoretical and experimental values of the on-shell V – P electromagnetic vertex couplings defined in Eq. (12). For $g_{VP\gamma}$ (th.) we give only the experimental errors coming from the decay constants $f_{P,V}$.

Finally, we introduce vertex couplings $g_{VP\gamma}$, which are just the on-shell V – P electromagnetic form factors:

$$\langle P(p_P) | J_\mu^{\text{EM}} | V(p_V, \lambda) \rangle \Big|_{(p_V - p_P)^2 = 0} = -g_{VP\gamma} \epsilon_{\mu\nu\rho\sigma} p_P^\nu p_V^\rho \epsilon_V^{(*)\lambda\sigma}. \quad (12)$$

The amplitude of the decay $P \rightarrow V\gamma$ or $V \rightarrow P\gamma$, depending on kinematics, is then obtained by contracting with the polarization vector of the photon and multiplying by $\sqrt{4\pi\alpha}$. The decay rates read

$$\Gamma(P \rightarrow V\gamma) = \frac{\alpha}{8} g_{VP\gamma}^2 \left(\frac{m_P^2 - m_V^2}{m_P} \right)^3, \quad \Gamma(V \rightarrow P\gamma) = \frac{\alpha}{24} g_{VP\gamma}^2 \left(\frac{m_V^2 - m_P^2}{m_V} \right)^3. \quad (13)$$

In Table 1 we give both the theoretical and experimental values of the couplings $g_{VP\gamma}$ for decays involving pions or kaons. The agreement between theoretical and experimental values shows that the ground states indeed dominate the spectral functions in (5), with corrections of order 10%. There are, however, three channels where the deviation from the experimental value is larger. The couplings in the ρ – π channels should be equal, those in the K^* – K channel differ by a factor of two, which is not quite supported by the data. Since the couplings are related by Clebsch-Gordan coefficients, we see no possibility to reconcile the data with our predictions and leave this “anomaly” as open question to the experimentalists. For $\phi \rightarrow \pi\gamma$ the deviation is more than 10%. This decay, however, is in our approach completely due to ϕ – ω non-ideal mixing and thus strongly suppressed compared to the other channels. It may also be influenced by other, usually negligible mechanisms, in particular ρ – ω mixing, which we have neglected here, so that we consider the agreement with the experimental coupling as still satisfactory.

3. Having gained some control over the radiative decays of vectors into π and K , we now turn to the η - η' system, which is our central point of interest in this study. Numerous possibilities, of which we can only quote a few [10], have been suggested to describe this system, with or without explicit mixing with extra “glueball” states. The discussion of all these approaches goes far beyond the scope of this letter, and we will content ourselves with shortly commenting in Sec. 6 on the elegant approach by Veneziano et al., Ref. [2]. For the time being, however, we focus on the more simple and pragmatic approach proposed in Ref. [1]. Following the Particle Data Group conventions, which differ from the ones originally used in Ref. [1], we reproduce here the key formulas in the current notation. In terms of the mixing angle θ and their singlet and octet components η_0 and η_8 , respectively, the physical states are decomposed as

$$|\eta\rangle = |\eta_8\rangle \cos \theta - |\eta_0\rangle \sin \theta, \quad |\eta'\rangle = |\eta_8\rangle \sin \theta + |\eta_0\rangle \cos \theta. \quad (14)$$

Traditional $\text{SU}_F(3)$ based analyses suggest either $\theta \simeq -10^\circ$ or $\theta \simeq -23^\circ$ from the quadratic and linear version, respectively, of the Gell–Mann–Okubo mass formula, whereas we leave θ as an open parameter, at least for the time being. Analogously to the pion decay constant f_π , Eq. (4), we define the decay constants⁶ f_8 and f_0 as coupling of η_8 and η_0 , respectively, to the divergence of the relevant axial-vector currents (where $f_8 = f_\pi \neq f_0$ in the $\text{SU}_F(3)$ limit):

$$\begin{aligned} \partial_\mu A_8^\mu &= \frac{2}{\sqrt{6}} \left(m_u \bar{u} i \gamma_5 u + m_d \bar{d} i \gamma_5 d - 2 m_s \bar{s} i \gamma_5 s \right), \\ \partial_\mu A_0^\mu &= \frac{2}{\sqrt{3}} \left(m_u \bar{u} i \gamma_5 u + m_d \bar{d} i \gamma_5 d + m_s \bar{s} i \gamma_5 s \right) + \frac{1}{\sqrt{3}} \frac{3}{4} \frac{\alpha_s}{\pi} G_{\mu\nu}^A \tilde{G}^{A\mu\nu}, \end{aligned} \quad (15)$$

where $G_{\mu\nu}^A$ is the gluonic field-strength tensor and $\tilde{G}_{\mu\nu}^A = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^{A\rho\sigma}$ its dual. Defining as in Ref. [1] the interpolating fields of η and η' as linear combinations of the axial-vector divergences, we thus have

$$\begin{aligned} \langle 0 | \partial_\mu A_8^\mu | \eta \rangle &= m_\eta^2 f_8 \cos \theta, & \langle 0 | \partial_\mu A_0^\mu | \eta \rangle &= -m_\eta^2 f_0 \sin \theta, \\ \langle 0 | \partial_\mu A_8^\mu | \eta' \rangle &= m_{\eta'}^2 f_8 \sin \theta, & \langle 0 | \partial_\mu A_0^\mu | \eta' \rangle &= m_{\eta'}^2 f_0 \cos \theta. \end{aligned} \quad (16)$$

Our essential assumption is that the u and d quark masses can be neglected in (15). This yields the following simple expressions for the matrix element of the strong anomaly over the vacuum and $\eta(\eta')$:

$$\begin{aligned} \langle 0 | \frac{3}{4} \frac{\alpha_s}{\pi} G \tilde{G} | \eta \rangle &= \sqrt{\frac{3}{2}} m_\eta^2 \left(f_8 \cos \theta - \sqrt{2} f_0 \sin \theta \right), \\ \langle 0 | \frac{3}{4} \frac{\alpha_s}{\pi} G \tilde{G} | \eta' \rangle &= \sqrt{\frac{3}{2}} m_{\eta'}^2 \left(f_8 \sin \theta + \sqrt{2} f_0 \cos \theta \right). \end{aligned} \quad (17)$$

⁶Note that by virtue of the “hard” non-conservation of A_0^μ , f_0 is scale-dependent as described in detail in the first reference of [2]; we will assume here that the momentum dependence can be neglected.

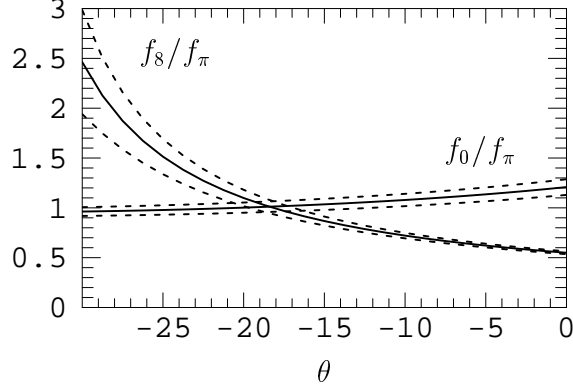


Figure 1: f_0/f_π and f_8/f_π , determined from $\eta \rightarrow 2\gamma$ and $\eta' \rightarrow 2\gamma$, as functions of the mixing angle.

We thus have three parameters to fix: f_0 , f_8 and θ . Two of them can be determined from the radiative widths of η and η' , which are given by:

$$\Gamma(\eta \rightarrow 2\gamma) = \frac{m_\eta^3}{96\pi^3} \alpha^2 \left(\frac{\cos \theta}{f_8} - \frac{2\sqrt{2} \sin \theta}{f_0} \right)^2, \quad (18)$$

$$\Gamma(\eta' \rightarrow 2\gamma) = \frac{m_{\eta'}^3}{96\pi^3} \alpha^2 \left(\frac{\sin \theta}{f_8} + \frac{2\sqrt{2} \cos \theta}{f_0} \right)^2. \quad (19)$$

As it was done in Ref. [1], we solve these relations for the decay constants and thus get f_0 and f_8 as functions of the mixing angle. As experimental input we use [9]

$$\Gamma(\eta \rightarrow 2\gamma) = (0.51 \pm 0.026) \text{ keV}, \quad \Gamma(\eta' \rightarrow 2\gamma) = (4.53 \pm 0.59) \text{ keV}, \quad (20)$$

where according to the suggestion in the full listings part of the Review of Particle Properties (Ref. [9], p. 1451) we have only retained the more recent data. In Fig. 1, f_0/f_π and f_8/f_π are plotted as functions of the mixing angle (solid lines) together with their experimental error (dashed lines). We find thus $f_8/f_\pi \geq 1$ for θ less than $\approx -15^\circ$ as expected from chiral perturbation theory. The precise value [11] $f_8/f_\pi = 1.25$, however, corresponds to a rather large⁷ mixing $\theta = -21.3^\circ$.

Having thus fixed f_0 and f_8 , we continue in the next section with the electromagnetic properties of η and η' , i.e. the vector meson decays, consider then in Sec. 5 their light quark matrix elements, to finish in Sec. 6 with their glue content.

4. We are now in a position to deal with the radiative vector meson decays involving η or η' . In Table 2 we list the decay channels together with the theoretical formulas for the couplings $g_{VP\gamma}$ and their experimental values. In Fig. 2 we then plot the theoretical and experimental values as functions of the pseudoscalar mixing angle. Since in our approach continuum contributions are neglected, the couplings should be slightly overestimated by

⁷In fact, Ref. [12] supplements the more traditional approaches to the 2γ decays of η and η' by taking into account continuum contributions, which just cancel the effect of the large $\text{SU}_F(3)$ breaking ratio $f_8/f_\pi = 1.25$. The result is then $\theta = -(17 \pm 2)^\circ$ with the experimental input data of Eq. (20). As we shall see in the following, this value is in excellent agreement with the results we find from the investigation of other decay channels.

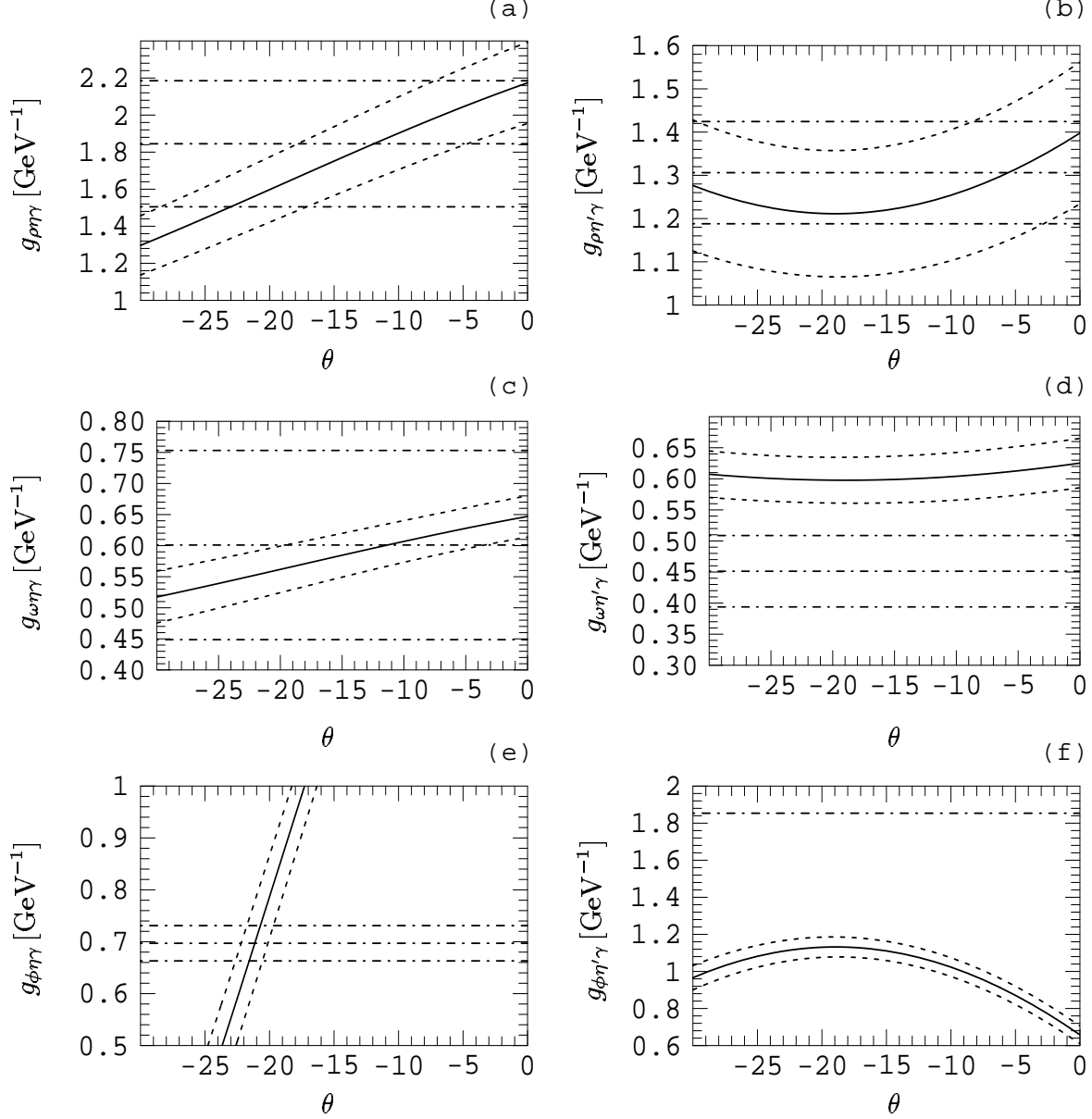


Figure 2: Theoretical and experimental values of the V - η, η' electromagnetic couplings as functions of the pseudoscalar mixing angle. Solid lines: theoretical predictions according to Table 2. Short-dashed lines: errors of the theoretical couplings coming from the experimental errors of f_0 and f_8 . Dashed-dotted lines: experimental couplings including errors (for $g_{\phi\eta'\gamma}$, plotted in (f), there exists only an experimental upper bound).

P	V	$g_{VP\gamma}$ in units $\frac{m_V}{f_V\pi^2}$	$g_{VP\gamma}$ (exp.)
η	ρ	$\frac{\sqrt{3}}{4f_8} \cos \theta - \sqrt{\frac{3}{8}} \frac{1}{f_0} \sin \theta$	$(1.85 \pm 0.34) \text{ GeV}^{-1}$
η'	ρ	$\sqrt{\frac{3}{8}} \frac{1}{f_0} \cos \theta + \frac{\sqrt{3}}{4f_8} \sin \theta$	$(1.31 \pm 0.12) \text{ GeV}^{-1}$
η	ω	$\frac{\cos \theta}{4f_8} (\sqrt{2} \cos \theta_V - \sin \theta_V) - \frac{\sin \theta}{2\sqrt{2}f_0} \sin \theta_V$	$(0.60 \pm 0.15) \text{ GeV}^{-1}$
η'	ω	$\frac{\cos \theta}{2\sqrt{2}f_0} \sin \theta_V + \frac{\sin \theta}{4f_8} (\sqrt{2} \cos \theta_V - \sin \theta_V)$	$(0.45 \pm 0.06) \text{ GeV}^{-1}$
η	ϕ	$-\frac{\cos \theta}{4f_8} (\cos \theta_V + \sqrt{2} \sin \theta_V) - \frac{\sin \theta}{2\sqrt{2}f_0} \cos \theta_V$	$(0.70 \pm 0.03) \text{ GeV}^{-1}$
η'	ϕ	$\frac{\cos \theta}{2\sqrt{2}f_0} \cos \theta_V - \frac{\sin \theta}{4f_8} (\cos \theta_V + \sqrt{2} \sin \theta_V)$	$< 1.85 \text{ GeV}^{-1}$

Table 2: Theoretical formulas and experimental values of the on-shell $V\text{--}\eta, \eta'$ electromagnetic vertex couplings defined in Eq. (12). In Fig. 2 the theoretical values are plotted as functions of θ . We use $\theta_V = 40.3^\circ$ for the $\phi\text{--}\omega$ mixing angle. Experimental values are taken from [9].

about 10 to 15%, as one may also infer from the cleanest channel $\omega \rightarrow \pi\gamma$ investigated in Sec. 2 (cf. Table 1). A look at Fig. 2(b) shows that this is not the case for $\eta' \rightarrow \rho\gamma$. In that channel, however, we expect a contamination of the experimental value from non-resonant $\pi\pi$ final states, which are produced by a completely different mechanism, which is related to the box-anomaly diagram (cf. e.g. [13]). For $\rho \rightarrow \eta\gamma$ and $\omega \rightarrow \eta\gamma$, Figs. 2(a) and (c), the experimental errors are too large to allow any serious restriction on θ . Within the error bars, however, our predictions agree with the experimental values. $g_{\omega\eta'\gamma}$, on the other hand, is very sensitive to the $\phi\text{--}\omega$ mixing angle; for instance, reducing θ_V by 1.5° brings the coupling within 20% agreement with the experimental data, i.e. just what we would expect.

As seen from the figures, the most stringent constraint on θ currently stems from the decay $\phi \rightarrow \eta\gamma$, which, as will turn out below, is only marginally consistent with the range of mixing angles found from other processes (remember however that we use 1σ experimental errors, and do not take into account theoretical uncertainties). The issue should be clarified when experimental errors will eventually shrink, and in particular we look forward to a measurement of the $\phi \rightarrow \eta'\gamma$ mode, for which our prediction is only a factor 2 below the current experimental upper bound.

5. Let us now turn to the investigation of other matrix elements of η and η' that can be probed in our approach. We start with the light quark matrix elements, which determine

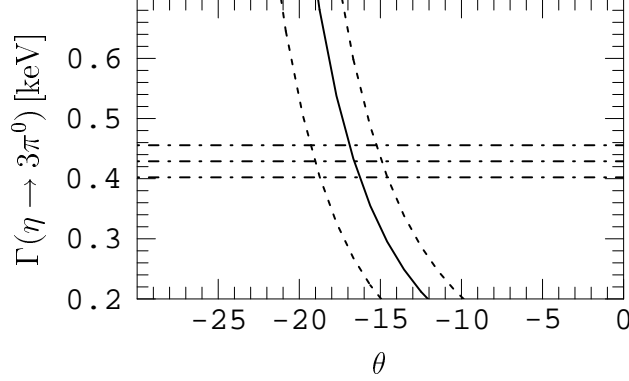


Figure 3: Theoretical (solid line) and experimental values (dashed-dotted lines, including experimental error) of the decay rate $\Gamma(\eta \rightarrow 3\pi^0)$ as function of the pseudoscalar mixing angle. The short dashes give the theoretical error of the rate, which is dominated by the error of the quark mass ratio.

the decay $\eta \rightarrow 3\pi$. As in Ref. [1] we find⁸:

$$\langle 0 | \bar{u}\gamma_5 u | \eta \rangle = -\frac{1}{\sqrt{2}} \frac{f_0 \cos \theta - \sqrt{2}f_8 \sin \theta}{\sqrt{2}f_0 \cos \theta + f_8 \sin \theta} \langle 0 | \bar{s}\gamma_5 s | \eta \rangle, \quad (21)$$

which yields the decay rate

$$\Gamma(\eta \rightarrow 3\pi^0) = \frac{\sqrt{3}}{4608\pi^2} m_\eta^3 (m_\eta - 3m_\pi)^2 \delta_\eta \left(\frac{m_d - m_u}{m_s} \right)^2 \frac{f_8^2 \cos^2 \theta}{f_\pi^6} \left(\frac{f_0 \cos \theta - \sqrt{2}f_8 \sin \theta}{\sqrt{2}f_0 \cos \theta + f_8 \sin \theta} \right)^2 \quad (22)$$

with a kinematical factor $\delta_\eta = 0.86$. The most crucial ingredient in that formula is the ratio of quark masses $r = (m_d - m_u)/m_s$. There exist several possibilities to fix r from next-to-leading order calculations in chiral perturbation theory, as discussed in [14], e.g. One of them is based essentially on the analysis of $\eta \rightarrow 3\pi$ in Ref. [15]. Since in that study η is necessarily treated as Goldstone boson, the value of r obtained that way is inconsistent with our analysis. For similar reasons, we also refrain from taking into account r determined from $\Gamma(\psi' \rightarrow J/\psi\eta)/\Gamma(\psi' \rightarrow J/\psi\pi)$. We thus rely on the determination of r from pseudoscalar mass relations, which according to Ref. [14] yield $r = 0.030 \pm 0.005$.

In Fig. 3 we plot the decay rate as function of θ . Although the agreement with the experimental rate is very good for $\theta \approx -17^\circ$, the theoretical errors are large and dominated by the uncertainty in the quark mass ratio. Actually the true theoretical error may be even larger and the rate be enhanced by final state interactions [14, 15].

In Ref. [1] we had also considered the decay $\eta' \rightarrow 3\pi$. We have however since argued [16] that this channel is considerably more complex and also receives contributions from the leading decay mode $\eta' \rightarrow \eta\pi\pi$ through η - π mixing. While this study is of great interest in itself and while a measurement of the charged mode $\eta' \rightarrow \pi^+\pi^-\pi^0$ could shed considerable light on the mechanism underlying $\eta' \rightarrow \eta\pi\pi$, it is of little help in determining the value of θ .

6. We finally turn to the most important issue concerning the η - η' system, namely the glue connection. Following the suggestion of Novikov et al., Ref. [17], the glue matrix

⁸Eq. (21) differs slightly from the corresponding Eq. (19) in Ref. [1], where we had made the simplification $f_0/f_8 \approx 1$.

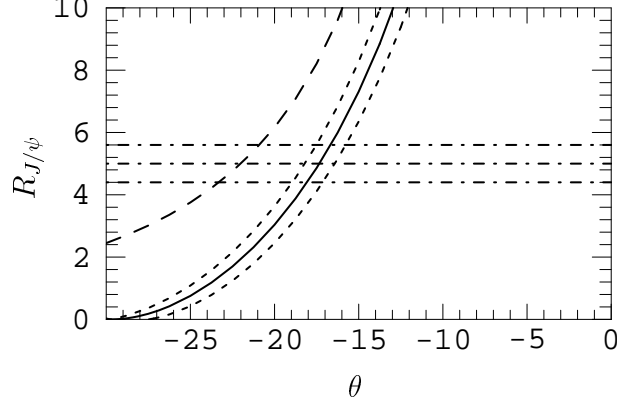


Figure 4: $R_{J/\psi} = \Gamma(J/\psi \rightarrow \eta'\gamma)/\Gamma(J/\psi \rightarrow \eta\gamma)$ as function of the mixing angle θ . Solid line: $R_{J/\psi}$ according to Eqs. (17) and (23) including experimental errors of f_0 and f_8 (short dashes). Long-dashed line: the naïve prediction (24). Dashed-dotted lines: experimental value with errors.

elements are tested in the ratio of the decay rates $\Gamma(J/\psi \rightarrow \eta(\eta')\gamma)$:

$$R_{J/\psi} \equiv \frac{\Gamma(J/\psi \rightarrow \eta'\gamma)}{\Gamma(J/\psi \rightarrow \eta\gamma)} = \left| \frac{\langle 0 | G\tilde{G} | \eta' \rangle}{\langle 0 | G\tilde{G} | \eta \rangle} \right|^2 \frac{(1 - m_{\eta'}^2/m_{J/\psi}^2)^3}{(1 - m_{\eta}^2/m_{J/\psi}^2)^3}. \quad (23)$$

Before turning to numerics, let us first comment shortly on the theoretical accuracy of the above equation. Actually the decay rates were calculated in exactly the same approximation we used in deriving the $g_{VP\gamma}$ given in the tables, namely dominance of the ground state and neglect of continuum contributions to the dispersion relations. It was also pointed out in Ref. [18] that the decay mechanism via the strong anomaly dominates only due to the smallness of the c quark mass and may be non-effective already in Υ decays, for which unfortunately no experimental data exist to date. It is for these reasons that we prefer to consider the *ratio* $R_{J/\psi}$ instead of the two decay rates separately: both radiative and continuum corrections are expected to cancel to some extent in $R_{J/\psi}$.

In Fig. 4 we plot $R_{J/\psi}$ according to Eqs. (17) and (23) and also the naïve prediction

$$R_{J/\psi} = \cot^2\theta \frac{(1 - m_{\eta'}^2/m_{J/\psi}^2)^3}{(1 - m_{\eta}^2/m_{J/\psi}^2)^3} \quad (24)$$

as functions of the mixing angle. As experimental input we use $R_{J/\psi}^{exp} = 5.0 \pm 0.6$ [9]. Our prediction (23) is rather sensitive to the mixing angle. If we estimate the remaining theoretical uncertainty in (23) as $\sim 10\%$, we obtain $\theta = -(17.3 \pm 1.3)^\circ$. It is clearly seen, on the other hand, that the naïve curve is less consistent with the data and requires a very large mixing angle. To understand the departure of our formula from the naïve prediction, we briefly evoke the elegant, but heavier formulation by Veneziano and collaborators [2]. These authors deal with a simplified model, where η_8 – η_0 mixing is neglected. To apply their approach to the physical situation requires the introduction of *three* initial fields (instead of two in our approach):

$$\phi_0^5 = \frac{1}{\sqrt{3}} (\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s), \quad \phi_8^5 = \frac{1}{\sqrt{6}} (\bar{u}\gamma_5 u + \bar{d}\gamma_5 d - 2\bar{s}\gamma_5 s), \quad Q = \frac{\alpha_s}{8\pi} G_{A\mu\nu} \tilde{G}^{A\mu\nu}. \quad (25)$$

The definition of interpolating fields corresponding to physical particles requires the orthonormalisation of these initial fields. While various procedures are possible (like introducing some "glue" Q in the definition of η' , as we did above when using the total divergences as interpolating fields), the choice of basis is essentially free, and the authors of Ref. [2] preferred to redefine the Q field, introducing G as below⁹. This leads to (we follow closely the notation of [2]):

$$\phi_\eta = b_8 \cos \theta \phi_8^5 - b_0 \sin \theta \phi_0^5, \quad \phi_{\eta'} = b_8 \sin \theta \phi_8^5 + b_0 \cos \theta \phi_0^5, \quad G = Q - \frac{W_{\Theta\eta'}}{W_{\eta'\eta'}} \phi_{\eta'} - \frac{W_{\Theta\eta}}{W_{\eta\eta}} \phi_\eta, \quad (26)$$

where the coefficients b_i and W_{ij} can be determined in terms of two-point functions. Expressing the relevant Ward–Takahashi identities in terms of the effective action and taking two derivatives with respect to the gluon fields (g , with indices omitted below), we obtain for the (non-anomalous) octet axial current:

$$\Gamma_{\phi_8^5 gg} = b_8 \sin \theta \Gamma_{\eta' gg} + b_8 \cos \theta \Gamma_{\eta gg} + X \Gamma_{G gg} = 0 \quad (27)$$

with X a computable coefficient. Clearly, if the proper vertex between G and two gluons could be neglected (in the way the similar coupling between G and two photons is legitimately neglected in Ref. [2]), we would recover the naïve result (24). This step is however not permitted since the proper vertex between G (whatever its physical realization) and two gluons is precisely expected to be large. It should thus not be surprising that we have to depart from Eq. (24), which is difficult to reconcile with the data, as already noted by Gilman and Kauffman in Ref. [10]. We will not dive deeper into this approach, except to mention that it is a clear generalization of ours (in our case the weight of glue in η' is directly fixed by our definition of its interpolating field, instead as resulting from a further diagonalization between the three states).

7. Having a rather comprehensive view of the η and η' system and its connection with glue, we now address other promising channels for future investigations, to wit D_s decays. Here two processes compete to produce $\eta(\eta')X$ final states: The first one is the decay $c \rightarrow s$, leading to a $\bar{s}s$ pair, which hadronizes to η or η' , while the second one can be described as $c\bar{s}$ annihilation into W and two gluons. It is suppressed by the Zweig-rule, but may in the present case gain importance through the large $\langle 0 | G\tilde{G} | \eta(\eta') \rangle$ matrix elements. We should thus expect both an enhancement of the $(\eta + \eta')X$ modes (unless the interference is destructive) and a large η'/η ratio, once the rates are corrected for phase-space and final state interactions.

If we ignore for the moment final state interactions, the simplest way to evaluate a possible enhancement is by direct comparison to similar D_0 decays. For instance the partial width for $D_0 \rightarrow K^- \rho^+$ (to which only the first diagram contributes) would, in absence of the glue contribution, be expected to be of the same order (after phase-space corrections) as the sum $D_s \rightarrow \eta \rho + D_s \rightarrow \eta' \rho$. Instead we find:

$$\frac{\Gamma(D_s \rightarrow \eta' \rho)(p_{\eta'})^{-3} + \Gamma(D_s \rightarrow \eta \rho)(p_\eta)^{-3}}{\Gamma(D_0 \rightarrow K^- \rho)(p_K)^{-3}} \left(\frac{m_{D_s}}{m_{D_0}} \right)^2 = 4.2 \pm 1.0, \quad (28)$$

⁹Note that the question if G is to be identified with any physical state, a glueball for instance, is left open in Ref. [2].

while for the branching ratios we find

$$\frac{\Gamma(D_s \rightarrow \eta' \rho)}{\Gamma(D_s \rightarrow \eta \rho)} \left(\frac{p_{\eta'}}{p_\eta} \right)^{-3} = 4.5 \pm 1.5, \quad (29)$$

which indicates both an enhancement of the decay rate and a large ratio for these important decay modes (of the order of 10% each). The situation is unfortunately less clear in the π associated modes, where the comparison with D_0 now gives:

$$\frac{\Gamma(D_s \rightarrow \eta' \pi)(p_{\eta'})^{-1} + \Gamma(D_s \rightarrow \eta \pi)(p_\eta)^{-1}}{\Gamma(D_0 \rightarrow K^- \pi)(p_K)^{-1}} = 1.6 \pm 0.4, \quad (30)$$

while

$$\frac{\Gamma(D_s \rightarrow \eta' \pi)}{\Gamma(D_s \rightarrow \eta \pi)} \left(\frac{p_{\eta'}}{p_\eta} \right)^{-1} = 3.0 \pm 1.1. \quad (31)$$

More sophisticated analyses of non-leptonic D_s decays were the subject of several studies, e.g. [19, 20, 21], which, however, all relied on the naïve picture of η - η' mixing without taking into account the anomaly. In Ref. [20] it was pointed out that in order to account for the data the non-leptonic rates involving η' need to be enhanced by an extra factor of up to three when compared with the corresponding D_0 decay amplitudes, even after including final state interactions. And the global fit done in Ref. [21] for all non-leptonic D and D_s decays including final state interactions, W exchange and annihilation contributions fails to explain the large branching ratios of D_s into $\eta' \pi$ and $\eta' \rho$. The enhancement needed is however well in line with our expectations from the extra Zweig-suppressed (gluon-mediated) amplitudes.

Very recently, the CLEO collaboration has measured the theoretically cleanest semileptonic channel and finds [22]

$$\frac{\Gamma(D_s \rightarrow \eta' e \nu)}{\Gamma(D_s \rightarrow \eta e \nu)} = 0.35 \pm 0.09 \pm 0.07. \quad (32)$$

Defining the relevant formfactors as

$$\langle \eta | \bar{s} \gamma_\mu c | D_s \rangle = f_+^\eta(q^2) (p_{D_s} + p_\eta)_\mu + f_-^\eta(q^2) (p_{D_s} - p_\eta)_\mu \quad (33)$$

with $q = p_{D_s} - p_\eta$ and analogously $f_\pm^{\eta'}$, CLEO's measurement yields

$$\frac{f_+^{\eta'}(0)}{f_+^\eta(0)} = 1.14 \pm 0.17 \pm 0.13, \quad (34)$$

where we assumed monopole form factors¹⁰ with a pole at $q^2 = m_{D_s^*}^2$. The naïve mixing model, on the other hand, predicts

$$\frac{f_+^{\eta'}(q^2)}{f_+^\eta(q^2)} = \frac{\cos \theta - \sqrt{2} \sin \theta}{\sin \theta + \sqrt{2} \cos \theta}, \quad (35)$$

which translates into $\theta = -(13.5 \pm 4.7)^\circ$ and is only marginally consistent with the determination from $\eta \rightarrow 2\gamma$ [12].

¹⁰We do not expect the anomalous contribution to change drastically the q^2 dependence of the form factors, which in $D \rightarrow K e \nu$ is a monopole experimentally.

As a temporary conclusion to this section, we would like to add that, while this sector is clearly difficult to investigate both theoretically and experimentally, it could prove very fruitful for our understanding of the rôle of anomalies and the importance of gluons. There is also the strong suggestion that D_s decays, not unlike the J/ψ radiative decays, constitute a "glue-rich" channel, where search for possible glueball states should be considered. We hope to come back to that issue in future work.

Let us finally remark that apart from understanding the structure of the η and η' particles, the size of their connection to glue is of importance to investigate possible glueball candidates [23]. In particular, the candidates $f_0(1500)$ from crystal barrel [24] and $f_0(1590)$ [25] are seen to decay more frequently into $\eta\eta'$ than into $\eta\eta$ (after phase-space correction), a ratio which we expect to be given by

$$\frac{\Gamma(f_0 \rightarrow \eta'\eta)}{\Gamma(f_0 \rightarrow \eta\eta)} = \left| \frac{\langle 0 | G\tilde{G} | \eta' \rangle}{\langle 0 | G\tilde{G} | \eta \rangle} \right|^2 \frac{p_{\eta\eta'}^{CM}}{p_{\eta\eta}^{CM}}, \quad (36)$$

where $p_{\eta\eta'}^{CM}$ are the respective centre of mass momenta. This relation is however highly momentum dependent, since we are close to the threshold, and obviously needs to be integrated over the particle width.

In conclusion, the above study gives a reasonably consistent description of the η - η' system for values of the mixing angle θ between -20 and -17 degrees, taking into account anomalies, as tested from various channels, respectively depicting eletromagnetic properties, light quark and gluon content. Our approach will be tested by a general improvement of the experimental measurements, including the $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ rates, and in particular by the observation of $\phi \rightarrow \eta'\gamma$. The investigation of "glue-rich" channels, namely the traditional radiative decays of J/ψ , but also, as advocated here, the D_s decays, will also yield a better understanding of this system and at the same time help in the search for glueballs, which might solve the puzzle.

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